Metacognition Learning (2006) 1: 85–97
DOI 10.1007/s11409-006-6584-x

EMPIRICAL STUDY

The effects of IMPROVE on mathematical knowledge, mathematical reasoning and meta-cognition

Zemira Mevarech · Shimon Fridkin

Received: 30 November 2005 / Revised: 7 December 2005 /
Accepted: 8 December 2005 / Published online: 22 March 2006

Abstract The purpose of the present study is to examine the effects of IMPROVE, a meta-cognitive instructional method, on students' mathematical knowledge, mathematical reasoning and meta-cognition. Participants were 81 students who studied a pre-college course in mathematics. Students were randomly assigned into one of two groups and groups were randomly assigned into one of two conditions: IMPROVE vs. traditional instruction (the control group). Both groups were exposed to the same learning materials, solved exactly the same mathematical problems, and were taught by the same experienced teacher. The IMPROVE students were explicitly trained to activate meta-cognitive processes during the solution of mathematical problems. The control group was exposed to traditional instruction with no explicit exposure to meta-cognitive training. Results indicate that the IMPROVE students significantly outperformed their counterparts on both mathematical knowledge and mathematical reasoning. In addition, the IMPROVE students attained significantly higher scores then the control group on the three measures of meta-cognition: (a) general knowledge of cognition; (b) regulation of general cognition; and (c) domain-specific meta-cognitive knowledge. The theoretical and practical implications are discussed.

Keywords metacognitive instruction · mathematical achievement · mathematical reasoning · knowledge of cognition · regulation of cognition

Mathematics is considered to be one of the most important subjects learned in school. TIMSS (Third International Mathematics and Science Study, 2000) reports that in most countries, more then one-fifth of the curriculum is devoted to the studies of mathematics (Mullis et al., 2000). Yet, many students face considerable difficulties in studying mathematics (e.g., TIMSS, 2000; PISA, 2003). Such evidence points to the need of developing effective instructional methods that have the potential to promote mathematics knowledge and reasoning.

Z. Mevarech (✉) · S. Fridkin
School of Education,
Bar-Ilan University, Israel
e-mail: mevarz@mail.biu.ac.il

The NCTM (National Council of Teachers of Mathematics) emphasizes that meaningful mathematical teaching has to enhance knowledge construction via problem solving, building connections, developing mathematical communication, and using various kinds of mathematical representations (NCTM, 2000). In particular, the NCTM (2000) enhances the importance of developing students' meta-cognition as a means for improving students' mathematical problem solving and reasoning. The present study is rooted in this approach, aiming to examine the effects of a meta-cognitive instructional method, called IMPROVE (Mevarech & Kramarski, 1997), on students' mathematical reasoning as well as on their general and domain specific meta-cognitive knowledge.

Meta-Cognition and Mathematics Education

Flavell (1979), the founder of research on meta-cognition, defines meta-cognition as "thinking about thinking". He distinguishes between two components of meta-cognition: (a) knowledge of cognitive processes and products; and (b) ability to control, monitor, and evaluate cognitive processes. Flavell (1979) argues that knowledge of cognition depends on the following inter-related components: meta-cognitive knowledge about self, the task and strategies; knowledge about how to use the strategies; and meta-cognitive experience. The later refers to one's feeling about being successful (or unsuccessful) in performing the task. According to this model, the meta-cognitive knowledge leads to strategy use which in turns affects the meta-cognitive experience that affects the acquisition of meta-cognitive knowledge and so on.

From the 1990s on, much of the research on meta-cognition has focused on its structure and specific components. For example, Schraw and Dennison (1994) describe eight meta-cognitive scales, classified into two factors: knowledge of cognition and regulation of cognition. Knowledge of cognition refers to declarative (the 'what'), procedural (the 'how'), and conditional (the 'when' and 'why') knowledge. *Regulation of cognition* refers to planning, monitoring, debugging, evaluating, and information managing. (The "Measurement" section describes the operational definitions of the scales).

While many studies analyzed meta-cognition independently on the domain in which the cognitive and meta-cognitive processes are activated, Kramarski and Mevarech (2003) suggest distinguishing between general and domain-specific meta-cognitive knowledge. They argue that the distinction between general and domain specific meta-cognitive processes is similar to the distinction made between general and domain-specific cognitive processes. According to Kramarski and Mevarech (2003) "*general meta-cognitive knowledge* is knowing about and being able to control and regulate problem solving processes regardless of the specific domain from which problems or tasks are drawn. *Domain-specific meta-cognitive knowledge* focuses on the unique features of each domain and therefore varies among domains" (p. 284). Montague and Bos (1990) who also examined meta-mathematics processes identified meta-cognitive processes that are activated before, during, and after solving mathematical problems. These studies have led researchers not only to construct more valid measures for assessing meta-cognition, but also to develop instructional methods that are based on meta-cognitive training.

Meta-Cognitive Instructional Methods and Mathematics Education

In the area of mathematics, long before Flavell (1979) introduced the concept of meta-cognition, Polya (1957) suggested to train students to activate what we call today meta-cognitive processes. According to Polya (1957), students have to be trained to comprehend the problem before solving it, plan the solution, implement the plan, and look backward (evaluation). In his well known book 'how to solve it?', Polya (1957) provided several examples showing how to use these 'heuristic strategies' in mathematics classrooms. Thus, using a different terminology, Polya (1957) in fact emphasized the importance of meta-cognitive training.

About three decades later, Schoenfefld (1985) video taped college students solving mathematics problems. Given these observations, Schoenfefld (1985) trained students to stop periodically during the solution of math problems and ask themselves: What am I doing right now? Why am I doing it? And how does it help me? Schoenfeld reported that college students who were trained to use these self-addressed questions improved mathematics achievement. Yet, Schoenfeld did not conduct a quasi-experimental study comparing the experimental and control groups.

On the basis of these studies, Mevarech and Kramarski (1997) suggest to enhance mathematical reasoning by training students to use a series of self-addressed meta-cognitive questions. The method is called IMPROVE, the acronym of all the teaching steps:

Introducing the new concepts,
Meta-cognitive questioning,
Practicing,
Reviewing,
Obtaining mastery,
Verification, and
Enrichment and remedial.

In IMPROVE, the teacher first introduces the new concepts, theorems, formula etc. to the whole class by modeling the meta-cognitive questioning technique. In the first studies on IMPROVE, students were trained to use three kinds of self-addressed meta-cognitive questions: comprehension questions, connection questions, and strategic questions. Later a fourth kind of meta-cognitive question was added: reflection question. *Comprehension questions* orient students to articulate the main ideas in the problem (e.g., What is the problem all about?). *Connection questions* lead students to construct bridges between the given problem and problems solved in the past (e.g., What are the similarities and differences between the given problem and problems you have solved in the past, and why?). *Strategic questions* refer to strategies appropriate for solving the problem (e.g., What strategies are appropriate for solving the problem, and why?). Finally, *reflection questions* guide students to look backward either during the solution process (e.g., Why am I stuck? What am I doing here?), or at the end (e.g., Does the solution make sense? Can I solve it differently?).

Following the teacher introduction, students practice problem solving by using the meta-cognitive self-addressed questions. The practicing could be implemented in cooperative settings or individually (see below). At the end of the lesson, the teacher reviews the main ideas and reduces difficulties by modeling the use of the self-addressed meta-cognitive questioning. Frequently, (e.g., every two weeks) the teacher evaluates students' progress and provides feedback followed by enrichment and remedial materials, as needed.

A series of studies examines the effects of IMPROVE on various measures of mathematics and science achievement. Mevarech and Kramarski (1997) analyzed the changes in mathematics achievement and reasoning of seventh graders who studied either under IMPROVE or traditional instruction. Results indicate that IMPROVE students significantly outperformed their counterparts in the non-treatment control group on algebra achievement. In particular, the examination focused on numerals, substitution, expression, word problems, and reasoning. Interestingly, the positive effects of IMPROVE were observed not only on topics taught immediately before the administration of the examination, but also on those introduced in the first semester, about 6 to 8 months prior to the administration of the final examination.

The initial findings of Mevarech and Kramarski (1997) raise a series of questions. First, to what extent are the effects of meta-cognitive instructional methods limited to "conventional" test-like problems? Will the effects be evident also on authentic mathematical tasks or on topics in other areas, such as science?[1] Second, what are the lasting effects of meta-cognitive instructional methods? Third, which modes of instruction are more appropriate for implementing meta-cognitive instructional methods: cooperative or individualized? Finally, to what extent activating the full set of meta-cognitive questioning is more effective then using each question in itself? The following paragraphs shortly review the studies that address these questions.

Exploring the effects of meta-cognitive instructional method on students' ability to solve various kinds of mathematical problems indicates that IMPROVE indeed improves students' ability to solve not only test-like problems, but also authentic tasks relating to everyday life (Kramarski, Mevarech, & Arami, 2002). In addition, current research shows that students exposed to IMPROVE are better able to construct mathematical models then their counterparts in the control group (Mevarech, Tabuk, & Sinai, in press).

The issue of the lasting effects of instructional methods in general, and meta-cognitive instructional methods in particular, is one of the most important issues in the educational research literature. If indeed students can transfer the knowledge they acquire under metacognitive instruction to new situations, then the method's advantages are evident beyond the immediate effects relating to 'here and now'. To address this issue, Mevarech and Kramarski (2003) compared mathematical achievement of students who at eighth grade studied under IMPROVE or traditional instruction, but at ninth grade were randomly reassigned to classrooms where all students studied under traditional instruction. Results indicate that the positive effects of IMPROVE were evident on the immediate and delayed examinations (Mevarech & Kramarski, 2003).

[1] Since the present study focuses on mathematics achievement and meta-cognitive knowledge, we will not review here studies that examine the effects of IMPROVE and IMPROVE-like methods on science education. The interested reader can look, for example, at Zion, Michalski, & Mevarech (2005).

Meta-cognitive instructional methods could be employed in different instructional settings, such as: individualized or cooperative. Kramarski and Mevarech (2003) investigated the effects of four instructional methods on students' mathematical reasoning and meta-cognitive knowledge. The instructional methods were cooperative or individualized learning combined with meta-cognitive training, and cooperative or individualized learning with no explicit meta-cognitive training. Results indicate that students exposed to meta-cognitive instruction outperformed their counterparts on mathematical achievement, various aspects of mathematical explanations, the solution of transfer tasks, and meta-cognitive knowledge. Mevarech (1999) also indicates that using the full set of self-addressed meta-cognitive questioning is more effective then using each kind of question by itself, particularly for lower achievers.

In all these studies, the main measures focus on students' ability to solve mathematical problems. Little attention has been given to the effects of meta-cognitive instruction on the development of students' meta-cognition. Furthermore, given that meta-cognition is not one entity that either one has or does not have (Schraw & Dennison, 1994) raises the question of the differential effects of meta-cognitive instruction on the various components of meta-cognition, including the general and domain-specific meta-cognitive knowledge, identified by Kramarski and Mevarech (2003). We hypothesized that IMPROVE will exert stronger effects on domain specific then on general meta-cognition because the later requires transfer of knowledge and skills from the original context in which they were introduced to new situations. The present study addresses these issues. It compares the effects of IMPROVE and traditional instructional methods on pre-college students' mathematical knowledge and mathematical reasoning, as well as on their general and domain-specific meta-cognitive knowledge.

Method

Participants

Participants were 81 pre-college Israeli students *(N = 54 males and 27 females; mean age = 22.33 (SD = 1.74) ranging from 19 to 27 years old)* who studied an advanced course in "mathematical functions." These students either failed or received a low score on the Israeli matriculation exam in mathematics ($M = 64.62$; $SD = 18.22$) and therefore could not be accepted to university studies, unless they improve their scores on the examination. Thus, the pre-college courses provide a second opportunity for those who failed in high-school.

The Instructional Unit

The present study focuses on one unit within the course on mathematics functions—maximum and minimum problems, also called optimization or extreme points. The maximum and minimum problems provide a set of restrictions for which an optimal solution is required (e.g., finde the maximum profit under a set of restrictions referring to certain expenses). Appendix A presents an example of such problem. This topic was chosen because of two reasons: (a) optimization problems are applicable to many subjects (e.g., economy, sciences, or "real life" situations) and therefore are of interest to most students. Second, the problems involve many mathematical areas including algebra, analysis, geometry, statistics, and even arithmetic. The course syllabus fits the requirements of the Israeli mathematical matriculation exam on this topic.

Measurements

The following measurements were employed in the present study: mathematical examinations (pretest and posttest), general meta-cognitive questionnaire, and domain specific meta-cognitive questionnaire. In addition, we analyzed the scores on the matriculation exam in mathematics—middle level (four point's credit) that students attained prior to the beginning of the study (at the end of high school.

Mathematical Examination

The mathematical examination is constructed of two parts (ten problems). The first part includes five open-ended mathematical problems that examined students' ability to solve maximum and minimum problems. Students were asked to solve the problems and specify all the solution steps. The second part presents five correct mathematical propositions. Students had to provide mathematical justifications for the propositions and explain their reasoning in writing. Examination time was three hours.

Two versions of the examination were constructed: one was used as a pretest, and the other as a posttest.

Scoring: The scores on each problem ranged from 0–10. Thus, the total scores ranged from 0–100. For the sake of simplicity, all scores, including those of mathematical knowledge and those of mathematical reasoning were transformed into percent correct answers.

Alpha Cronbach reliability scores were .79 and .69, on the pretest and posttest, respectively.

Meta-Cognitive Questionnaires

Two kinds of meta-cognitive questionnaires were used in the present study: general and domain specific.

General Meta-Cognition Questionnaire (GMC-Q)

The meta-cognitive awareness inventory (MAI) developed by Schraw and Dennison (1994) was used to assess students' general meta-cognition. The MAI is a 52 item questionnaire composed of two parts: knowledge of cognition, and regulation of cognition. The first part includes 17 items referring to meta-cognitive knowledge: declarative, procedural, and conditional. According to Schraw and Dennison (1994), "*declarative knowledge* refers to knowledge about one's skills, intellectual resources, and abilities as a learner. *Procedural knowledge* refers to knowledge about how to implement learning procedures (e.g., strategies). *Conditional knowledge* refers to knowledge about when and why to use learning procedures" (p. 474).

The second part of MAI includes 35 items referring to regulation of cognition: planning, information management, monitoring, debugging, and evaluation. The operational definitions of these categories are as follows:

> "*Planning*: planning, goal setting, and allocating resources prior to learning;
> *Information management*: skills and strategy sequences used on-line to process information more efficiently;
> *Monitoring*: assessment of one's learning or strategy use;
> *Debugging*: strategies used to correct comprehension and performance errors;
> *Evaluation*: analysis of performance and strategy effectiveness after a learning episode" (Schraw & Dennison, 1994, pp. 474–475).

Scoring: Each item was scored on a five-point Likert type scale ranging from never (1) to always (5). Alpha Cronbach equals to .87.

Domain Specific Meta-Cognitive Knowledge Questionnaire (DSMK-Q)

A 24 item questionnaire adapted from the study of Montague and Bos (1990) assessed students' meta-cognitive knowledge in the area of solving maximum and minimum problems. The questionnaire refers to the use of strategies prior to, during, and after the solution of such problems.

Scoring: Each item was scored on a five-point Likert type scale ranging from never (1) to always (5). Alpha Cronbach equals .85.

Treatments

All students studied the course mathematical functions. As indicated, the instructional unit on which the study focuses is maximum and minimum problems. The unit was taught for 12 hours a week during one month (about 50 hours). All students studied the same problems and used the same learning materials. One teacher taught all students. The teacher had more than ten years of experience in teaching advanced courses in mathematics.

Students were randomly assigned into one of two groups and groups were randomly assigned to conditions. One group (N = 38) was exposed to IMPROVE and the other (N = 43) to traditional learning instruction.

IMPROVE–Experimental Group

The IMPROVE method was implemented as follows. The teacher first explained the advantages of using the self-addressed meta-cognitive questioning technique. Then, he modeled the use of the 'comprehension questions' in solving an optimization problem and students practiced the comprehension questions while solving the maximum and minimum problems. The same procedure was repeated for introducing and practicing the connection questions, strategic question, and reflection questions. Each type of self-addressed meta-cognitive questioning was practiced for 3–6 sessions, all together 20 sessions. The rest of the time, students continued to use the four kinds of meta-cognitive questioning during the solution of mathematics problems. Students practiced the problems in individualized settings and the teacher provided assistance as needed. At the end of the session, the teacher reviewed the solution of the mathematical problems by modeling the meta-cognitive questioning.

Traditional Instruction-Control Group

The control group was exposed to the traditional method of instruction, in which the teacher introduced the new concepts to the whole class, and then students practiced the problems relating to the new concepts. As in the experimental group, also in the control group, during the practicing, the teacher provided assistance to students as needed.

Procedure

Prior to the beginning of the study, all students were administered the mathematical examination (pretest) followed by the two meta-cognitive questionnaires: the general and the domain-specific. In addition, students fulfilled a short information questionnaire regarding their age, gender, and mathematical score on the matriculation examination. Initial comparisons of the experimental and control groups indicate no significant differences between the groups on age (M = 22.44 and 22.21 SD = 1.623 and 1.833, for IMPROVE and control groups, respectively; $F < 1.00$; $p = .549$), mathematics matriculation scores (M = 63.42 and 65.67; SD = 20.78 and 15.80, for IMPROVE and control, respectively; $F < 1.00$; $p = .582$), and gender (30 and 24 boys in IMPROVE and control groups, respectively; chi square = 3.87, critical value = 3.84; $p = .05$).

After the pre-testing, students started to study the solution of maximum and minimum mathematics problems according to the condition to which they were assigned: IMPROVE or traditional instruction. As indicated, the duration of the study was one month.

At the end of the study, all students were administered the posttest and the two meta-cognitive questionnaires: the general and the domain.

Results

The data were analyzed as follows. First, one-way ANOVA examines the differences between IMPROVE and control groups on the pretest scores. Then, one-way ANCOVA compares the mean scores on the posttest, controlling for differences on the corresponding pretest. ANCOVA was employed after checking the pre-requisites for running it. All pre-requisites were attained.

Mathematics Achievement

Table 1 presents the mean scores and standard deviations on mathematics achievement by time and treatment. According to Table 1, although no significant differences were found between the two groups on the achievement mean scores prior to the beginning of the study [($F(1,79) < 1.00$, $p > .05$)], ANCOVA indicates significant differences between conditions at the end of the study, controlling for initial differences between conditions [($F(1,78) = 5.21$, $p < .05$)].

Further analyses attempt to examine the effects of IMPROVE on mathematical knowledge and mathematical reasoning. One-way ANOVA and ANCOVA indicate that although no significant differences between conditions were found prior to the

Table 1 Students' mean scores and standard deviations on mathematics achievement by time and treatment

		IMPROVE	Control
Total Score			
Pretest	M	63.55	66.70
	SD	23.14	23.58
Posttest	M	75.08	66.91
	SD	18.74	24.18
Mathematical knowledge			
Pretest	M	63.42	66.61
	SD	23.20	24.19
Posttest	M	74.50	62.62
	SD	19.20	24.72
Mathematical reasoning			
Pretest	M	67.26	67.93
	SD	21.85	25.07
Posttest	M	75.08	66.91
	SD	18.74	24.18

beginning of the study [(both $F(1,79) < 1.00$; $p > .05$)], significant differences were found at the end of the study [($F(1,78) = 10.14$ and 15.45; $p = .002$ and .001 on mathematical knowledge and mathematical reasoning, respectively)]. According to Table 1, at the end of the study, IMPORVE students significantly outperformed the control group on both mathematical knowledge and mathematical reasoning.

General Meta-Cognition

Table 2 presents the mean scores and standard deviations on general meta-cognition (knowledge and regulation) by time and treatment. The analyses indicate significant differences between conditions prior to the beginning of the study on both knowledge of cognition and regulation of cognition [($F(1,79) = 4.91$ and 6.75, on knowledge and regulation, respectively; both $p < .05$)]. ANCOVA, indicate, however, that at the end of the study, the differences between the groups largely increased [($F(1, 78) = 16.985$ and 18.464, on knowledge and regulation, respectively, both $p < .01$)].

Further analyses examined MAI scales under each condition. Table 3 presents the mean scores and standard deviations on each scale by time and treatment.

Table 2 Students' mean scores and standard deviations on general meta-cognition by time and treatment

		IMPROVE	Control
Knowledge of cognition–total scores			
Pretest	M	3.46	3.31
	SD	.26	.33
Posttest	M	4.04	3.76
	SD	.29	.28
Regulation of cognition–total scores			
Pretest	M	3.57	3.42
	SD	.28	.25
Posttest	M	4.13	3.82
	SD	.32	.26

Table 3 Students' mean scores and standard deviations on each MAI scale by time and treatment

		IMPROVE	Control
Knowledge of cognition			
Declarative knowledge			
Pretest	M	3.51	3.42
	SD	.33	.38
Posttest	M	4.10	3.83
	SD	.34	.31
Procedural knowledge			
Pretest	M	3.43	3.26
	SD	.64	.60
Posttest	M	4.01	3.72
	SD	.68	.50
Conditional knowledge			
Pretest	M	3.41	3.18
	SD	.39	.42
Posttest	M	3.98	3.67
	SD	.35	.32
Regulation of cognition			
Planning			
Pretest	M	3.58	3.47
	SD	.46	.39
Posttest	M	4.14	3.88
	SD	.41	.32
Information management			
Pretest	M	3.55	3.32
	SD	.36	.37
Posttest	M	4.10	3.75
	SD	.38	.33
Monitoring			
Pretest	M	3.70	3.50
	SD	.28	.25
Posttest	M	4.17	3.85
	SD	.44	.45
Debugging			
Pretest	M	3.46	3.38
	SD	.55	.47
Posttest	M	4.13	3.76
	SD	.47	.45
Evaluation			
Pretest	M	3.54	3.48
	SD	.50	.41
Posttest	M	4.12	3.88
	SD	.42	.35

According to Table 3, prior to the beginning of the study, no significant differences were found on any scale [(F(1,79) values range from .33 to 3.78, all $p > .05$]; except conditional knowledge on which initial differences between groups were statistically significant [($F(1,79) = 6.19$, $p < .02$)]. Yet, at the end of the study, the differences on **all** scales were statistically significant [(F(1, 78) values ranged from 5.06 to 22.94, all $p < .01$)]. According to Table 3 at the end of the study, IMPROVE students attained higher scores on all MAI scales compared to the control group.

Table 4 Students' mean scores and standard deviations on domain specific meta-cognitive knowledge by time and treatment

		IMPROVE	Control
DSMK–Total Scores			
Pretest	M	3.58	3.53
	SD	.40	.29
Posttest	M	4.27	3.62
	SD	.25	.30
Using strategies prior to problem solving			
Pretest	M	3.86	3.89
	SD	.61	.45
Posttest	M	4.42	3.90
	SD	.41	.44
Using strategies during problem solving			
Pretest	M	3.61	3.55
	SD	.52	.57
Posttest	M	4.23	3.64
	SD	.36	.45
Using strategies at the end of problem solving			
Pretest	M	3.45	3.38
	SD	.42	.35
Posttest	M	4.23	3.50
	SD	.26	.38

Domain-Specific Meta-Cognitive Knowledge

Table 4 presents the mean scores and standard deviations by time and treatment on the domain-specific meta-cognitive knowledge (DSMK). According to Table 4, prior to the beginning of the study, no significant differences were found between IMPROVE and control groups on DSMK [$(F(1,79) = .45, p = .504)$], but significant differences were found at the end of the study controlling for prior scores [$(F(1,78) = 14.58, p < .001)$]. Analyzing the differences between conditions on the use of strategies in solving maximum and minimum problems prior to, during, and at the end of the study, indicates no significant differences between conditions on any DSMK component prior to the beginning of the study [(all $F(1,79)$ values < 1.00; p values > .05)], but at the end of the study significant differences were found between the groups on all components [($F(1,78)$ values ranged from 68.87 to 146.80, all $p < .0001$)].

Discussion

The major purpose of the present study is to examine the effects of a meta-cognitive instructional method, called IMPROVE, on students' mathematical achievement and meta-cognition. In the present study, mathematics achievement refers to both mathematics knowledge and mathematics reasoning. Findings show that IMPROVE students outperformed their counterparts on both measures. These findings are in line with Schoenfeld's (1985) study showing the changes in college students' mathematical reasoning under a meta-cognitive instructional method. The findings also enlarge previous research on IMPROVE which mainly focused on junior high school students (e.g., Kramarski & Mevarech, 2003; Mevarech & Kramarski, 1997; Mevarech, 1999).

In contrast to previous research in which learning mathematics was compulsory, in the present study students under both conditions elected the course and were highly motivated to attain high scores on the final examination. Hence, the lower achievement level of the control group cannot be attributed to lack of motivation. Furthermore, since both groups studied exactly the same problems, used the same learning materials, and had the same teacher, there is reason to suppose that the differences between the two conditions are due to the meta-cognitive training.

The findings of the present study further show that IMPROVE students developed a higher level of meta-cognition compared to their counterparts under the control group. The effects of IMPROVE were evident on all eight scales of general meta-cognitive awareness, including knowledge of cognition (declarative, procedural, and conditional) and regulation of cognition (planning, monitoring, debugging, evaluating, and information managing). IMPROVE's positive effects were also evident on all three components of the domain-specific meta-cognitive knowledge (DSMK), including the use of strategies prior to, during, and immediately after solving problems. These findings should not surprise us because IMPROVE students were exposed to meta-cognitive training and therefore became more aware of the meta-cognitive processes. The fact that in the present study students practiced the use of the self-addressed meta-cognitive questioning for about 12 hours a week for one month may strengthen the effects of IMPROVE on the meta-cognitive and cognitive domains.

The findings of the present study raise several questions for future research. First, all measures used in the present study were paper-and-pencil. There is an urgent need to conduct observations under meta-cognitive instructional methods. Direct observations, field-notes, video tapping, interviews with students and teachers may throw light on who benefits form IMPROVE and why.

Another issue that merits future research is the differential effects of IMPROVE on boys and girls. Since IMPROVE is a highly verbalized instructional method, one may argue that it would exert stronger effects on girls' achievement and meta-cognition compared to boys. This issue is particularly interesting given the lower scores that girls frequently attain on mathematics examinations.

Finally, there is a need, both theoretical and practical, to examine the effects of IMPROVE and other meta-cognitive instructional methods on students from different countries. The international studies (e.g., PISA, 2003) indicate that examining mathematical achievement should be followed by other measures, including those that focus on meta-cognition. Since the studies of mathematics is highly emphasized in the global village, and since the need to develop effective means to promote students' mathematical reasoning and meta-cognitive knowledge is also widely recognized, it seems necessary to examine the implementation of IMPROVE and IMPROVE-like methods in various educational settings.

Appendix: Examples for the Two Kinds of Problems in the Mathematical Examination

An Optimization Problem

A merchant sells goods for 355 NIS a day. His expanses are on the first day 1 NIS, on the second day 2 NIS, on the third day 4 NIS etc. After how many days will his profit be maximum? Please describe all stages of the solution, including the calculations, and explain your reasoning.

Justification of Mathematical Proposition

From all triangles with perimeter p, the equilateral triangle has the biggest area. Please justify this proposition.

References

Flavell, J. H. (1979). Meta-cognitive and cognitive monitoring: A new area of cognitive developmental inquiry. *American Psychologist, 31*, 906–911.

Kramarski, B., & Mevarech, Z. R. (2003). Enhancing mathematical reasoning in the classroom: The effects of cooperative learning and meta-cognitive training. *American Educational Research Journal, 40*, 281–310.

Kramarski, B., Mevarech, Z. R., & Arami, M. (2002). The effects of meta-cognitive training on solving mathematical authentic tasks. *Educational Studies in Mathematics, 49*, 225–250.

Mevarech, Z. R. (1999). Effects of meta-cognitive training embedded in cooperative settings on mathematical problem solving. *The Journal of Educational Research, 92*, 195–205.

Mevarech, Z. R. & Kramarski, B. (1997). IMPROVE: A multidimentional method for teaching mathematics in heterogeneous classrooms. *American Educational Research Journal*, 34, 365–394.

Mevarech, Z. R., & Kramarski, B. (2003). The effects of worked-out examples vs. meta-cognitive training on students' mathematical reasoning. *British Journal of Educational Psychology, 73*, 449–471.

Mevarech, Z.R., Tabuk, A., & Sinai, O. (accepted for publication). Metacognitive instruction in mathematics classrooms: Effects on the solution of different kinds of problems.

Montague, M., & Bos, C. S. (1990). Cognitive and meta-cognitive characteristics of eighth-grade students' mathematical problem solving. *Learning and Individual Differences, 2*, 371–388.

Mullis, I. V. S., Martin, M. O., Gozalez, E. J., Gregory, K. D., Gaden, R. A., O'Conner, K. M., Chrostowski, S. J., & Smith, T. A. (2000). *TIMSS: International Mathematics Report. Findings form IEA's Repeat of the Third International Mathematics and Science Study at Eighth Grade*. Boston College.

National Council of Teachers of Mathematics (NCTM) (2000). *Principles and standards for school mathematics*. Reston, VA.

PISA (2003). *Literacy skills for the world of tomorrow: Further results from PISA 2000*. OECD. Paris.

Polya, G. (1957). *How to solve it? 2nd ed.* NJ: Princeton University Press.

Schoenfefld, A. H. (1985). *Mathematical problem solving*. San Diego, CA: Academic Press.

Schraw, G. & Dennison, R. S. (1994). Assessing meta-cognitive awareness. *Contemporary Educational Psychology, 19*, 460–475.

Zion, M., Michalski, T., & Mevarech, Z. R. (2005). The effects of meta-cognitive instruction embedded within asynchronous learning network on scientific inquiry skills. *International Journal of Science Education, 27*, 957–983.